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On the theory of the relativistic cross-sections for stimulated bremsstrahlung on an arbitrary electrostatic potential in the strong electromagnetic field

Theory of stimulated bremsstrahlung

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Abstract. On the base of relativistic generalized eikonal approximation wave function the multiphoton cross-sections of a Dirac particle bremsstrahlung on an arbitrary electrostatic potential and strong laser radiation field are presented. In the limit of the Born approximation the ultimate analytical formulas for arbitrary polarization of electromagnetic wave have been obtained and numerically analyzed.

PACS. 34.80.Qb Laser-modified scattering – 32.80.Wr Other multiphoton processes

1 Introduction

The appearance of superpower ultrashort laser pulses exceeding the intensities 10^{18} W/cm^2 in optical region or 10^{16} W/cm^2 in near-infrared region, when the energy of the interaction of electron with the field over a wavelength exceeds the electron rest energy opens new field of experimental studies of laser-assisted free-free and bound-free transitions. Generally, the interaction of such fields with the electrons in the presence of a third body makes available the revelation of many nonlinear relativistic quantum-electrodynamic phenomena such as production of electron-positron pairs from intense light field [1], nonlinear Compton scattering [2] and etc. As a third body can serve ion and in the superintense laser fields one can observe electron-positron pairs production and multiphoton stimulated bremsstrahlung (SB). Note that among the first experiments on the observation of multiphoton exchange between the free electrons and laser radiation was the stimulated bremsstrahlung [3]. Many papers have been devoted to the theoretical investigation of the electron-ion scattering processes in the presence of a laser field using quantal as well as classical considerations. These investigations have been carried out mainly within the framework of non-relativistic quantum theory (see, *e.g.*, [4]). On the other hand, in the current superintense laser fields the state of electron becomes relativistic already at the distances $l < \lambda$ (λ is the wavelength of a laser radiation) independent on its initial state. Hence, relativistic treatment of SB process is of great interest. Only a few papers devoted to the relativistic consideration of SB. The first nonrelativistic treatment of SB in the Born approximation has been carried out analytically in [5] and then this approach has been extended to the relativistic domain [6] where main emphasis made to the total cross-section in various limits. Relativistic effects in the SB corresponding to the relativistic first Born approximation with the comparison to a spinless and a nonrelativistic treatment for the circularly polarized electromagnetic (EM) wave have been investigated in [7,8]. Low-frequency approximation for SB in the relativistic case has been carried out in [9]. In the scope of classical theory the effect of an intense EM wave on the dynamics of SB and non-linear absorption of intense laser radiation by an electron beam due to SB have been carried out in [10,11].

In the papers [12,13] so-called generalized eikonal approximation (GEA) has been developed in the relativistic quantum theory of elastic scattering of Dirac particle on an arbitrary electrostatic field [12] and for the SB in the presence of an external strong EM radiation field [13]. These wave functions enable to leave the framework of ordinary eikonal approximation for elastic as well as for inelastic scattering [14]. The ordinary eikonal approximation is not applicable beyond the interaction region ($z \ll pa^2/\hbar$, where z is the coordinate along the direction of initial momentum $\mathbf{p}$ of the particle and a is the characteristic size of the interaction region). The GEA wave function is applicable in both quantum and quasiclassical limits, *i.e.*, connects the particle wave functions of the Born and ordinary eikonal approximations. In the present paper the relativistic cross-sections of SB in the scope of the above-mentioned GEA approximation are obtained. Relatively simple formulas for the transition amplitudes and cross-sections for the Dirac-particle scattering in the

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presence of an arbitrary polarized plane electromagnetic wave in the limit of the Born approximation are obtained.

The organization of the paper is as follows: in Section 2 the analytic expressions for differential cross-sections of the SB on an arbitrary electrostatic potential are obtained with the help of the dynamic GEA wave function and in the limit of Born approximation. The spin interaction is considered as well. In Section 3 we consider the multiphoton cross-sections of SB on screening Coulomb potential. Finally, conclusions are given in Section 4.

2 Multiphoton cross-sections of stimulated bremsstrahlung

The knowledge of solution of evolution equation for a Dirac particle interacting with the electrostatic and electromagnetic fields allows to calculate the scattering amplitude which takes into account the interaction with both potential and EM fields simultaneously. In the case when the wave function describes the particle states only in the region where the potential energy $U(\mathbf{r})$ is not zero it is impossible to determine the scattering amplitude by the asymptote of the wave function [17]. However, the scattering amplitude can be defined by the Green function formalism [18]. As far as the wave function in the GEA describes the particle states within the range of the scattering field and at asymptotic large distances, both approaches with such a wave function are applicable. In this paper we shall consider both approaches with an emphasis on asymptotic one.

We assume the EM wave to be quasimonochromatic and of an arbitrary polarization with the vector potential

$$\mathbf{A}(\varphi) = A_0(\varphi)(\widehat{\mathbf{e}}_1 \cos\varphi + \widehat{\mathbf{e}}_2 \zeta \sin\varphi), \tag{1}$$

where $A_0(\varphi)$ is a slow varying amplitude of the vector-potential of the plane EM wave $\mathbf{A}(t,\mathbf{r})$ with the phase $\varphi = kx$, $k = (\omega, \mathbf{k})$ is the four-wave vector of EM wave with frequency ω (here we set relativistic units $\hbar = c = 1$), $\widehat{\mathbf{e}}_1\widehat{\mathbf{e}}_2 = \widehat{\mathbf{e}}_1\mathbf{k} = \widehat{\mathbf{e}}_2\mathbf{k} = 0$, $|\widehat{\mathbf{e}}_1| = |\widehat{\mathbf{e}}_2| = 1$, and $\arctan\zeta$ is the polarization angle.

The state of the particle in EM wave field is characterized by the average four-kinetic momentum (quasimomentum) $\mathit{\Pi} = (\mathit{\Pi}_0, \mathbf{\Pi})$ defined via free electron four-momentum $p = (\varepsilon, \mathbf{p})$ and relativistic invariant parameter of the wave intensity Z by the following equation

$$\mathit{\Pi} = p + kZ(1+\zeta^2); \quad Z = \frac{e^2\overline{A}_0^2}{4kp}, \tag{2}$$

($\overline{A}_0$ is the averaged value of the $A_0(\varphi)$, e is the particle charge) with corresponding Volkov wave function

$$\mathit{\Psi}^V_{\mathit{\Pi},\mu} = \frac{1}{\sqrt{2\mathit{\Pi}_0}} f_\mathrm{V}(\varphi)\exp\left[\mathrm{i}S_\mathrm{V}(x)\right], \tag{3}$$

where

$$S_\mathrm{V}(x) = -\mathit{\Pi}x + \alpha\left(\frac{\mathbf{\Pi}}{k\mathit{\Pi}}\right)\sin[\varphi - \theta(\mathbf{\Pi})] - \frac{Z(1-\zeta^2)}{2}\sin 2\varphi \tag{4}$$

is the classical action of charged particle in EM wave field (1) and

$$f_\mathrm{V}(\varphi) = u_p^\mu - \frac{eA_0(\varphi)(\gamma k)}{2(k\mathit{\Pi})}\left[(\gamma\mathbf{e}_1)\cos\varphi + \zeta(\gamma\mathbf{e}_2)\sin\varphi\right)]\, u_p^\mu \tag{5}$$

is the bispinor amplitude, where $\gamma = (\gamma_0, \gamma)$ are the Dirac matrices, u_p^μ is the bispinor amplitude of a free particle with polarization μ, four-momentum p and mass m, $\overline{u}_p^\mu u_p^\mu = 2m$, $\overline{u}_p = u_p^\dagger\gamma_0$. The products like bx, bk, γk are relativistic scalar products

$$bx = b^0x^0 - \mathbf{b}\cdot\mathbf{x},$$

and the quantities $\alpha(\mathbf{b})$, $\theta(\mathbf{b})$ as functions of any vector $\mathbf{b}$ are defined by the relations

$$\alpha(\mathbf{b}) = e\overline{A}_0\sqrt{(\mathbf{b}\widehat{\mathbf{e}}_1)^2 + \zeta^2(\mathbf{b}\widehat{\mathbf{e}}_2)^2}, \tag{6}$$

$$\theta(\mathbf{b}) = \arctan\left(\frac{\mathbf{b}\widehat{\mathbf{e}}_2}{\mathbf{b}\widehat{\mathbf{e}}_1}\zeta\right)\cdot \tag{7}$$

The wave function of Dirac particle in GEA describing induced scattering on an arbitrary electrostatic field in the presence of strong EM radiation field (1) has the following form [13]

$$\mathit{\Psi}_{\mathit{\Pi},\nu} = \frac{1}{\sqrt{2\mathit{\Pi}_0}}\left(f_\mathrm{V}(\varphi) + f_1(x)\right)\exp\left[\mathrm{i}S_\mathrm{V}(x) + \mathrm{i}S_1(x)\right], \tag{8}$$

where $f_1(x)$ and $S_1(x)$ are the action and bispinor amplitude which describe the simultaneous effect of both the scattering and EM radiation fields on the particle state and can be represented in the following form

$$S_1(t,\mathbf{r}) = \frac{\mathrm{i}}{4\pi^3}\sum_{n=-\infty}^{\infty}\int\frac{\exp\{\mathrm{i}Q_n(\mathbf{r},\mathbf{q},\varphi)\}\,\widetilde{U}(\mathbf{q})}{\mathit{\Delta}_n(\mathbf{q}) - \mathrm{i}0} \times\left[\varepsilon D_n - \omega\alpha\left(\frac{\mathbf{p}}{kp}\right)D_{1,n}(\theta(\mathbf{p})) + \omega Z D_{2,n}\right]\mathrm{d}\mathbf{q}, \tag{9}$$

$$f_1(t,\mathbf{r}) = \frac{1}{(2\pi)^3}\sum_{n=-\infty}^{\infty}\int\frac{F_n(\varphi,\mathbf{q})\mathrm{d}\mathbf{q}}{\mathit{\Delta}_n(\mathbf{q}) - \mathrm{i}0}\cdot \tag{10}$$

Here

$$Q_n(\mathbf{r},\mathbf{q},\varphi) = -n\varphi + \mathbf{qr} + \alpha_1(\mathbf{q})\sin\left[\varphi - \theta_1(\mathbf{q})\right] - \alpha_2(\mathbf{q})\sin 2\varphi + \theta_1(\mathbf{q})n, \tag{11}$$

$$\mathit{\Delta}_n(\mathbf{q}) = \mathbf{q}^2 + 2\mathbf{pq} + 2Z\mathbf{kq} - 2n(kp - \mathbf{kq}), \tag{12}$$

and $\widetilde{U}(\mathbf{q}) = \int U(\eta)\exp(-\mathrm{i}\mathbf{q}\cdot\eta)\mathrm{d}\eta$ is the Fourier transform of the function $U(\mathbf{r})$. The bispinor $F_n(\varphi,\mathbf{q})$ is defined as follows

$$F_n(\varphi,\mathbf{q}) = \exp\{\mathrm{i}Q_n(\mathbf{r},\mathbf{q},\varphi)\}\,\widetilde{U}(\mathbf{q})\Bigg\{D_n(\gamma\mathbf{q})\gamma_0 + \frac{(\gamma k)(\gamma\mathbf{D})(\gamma\mathbf{q})\gamma_0}{2(kp-\mathbf{k}\cdot\mathbf{q})} + ZD_{2,n}\frac{(\mathbf{kq})(\gamma k)\gamma_0 - \omega(\gamma k)(\gamma\mathbf{q})}{kp-\mathbf{kq}} + \frac{1}{2kp}\left[\frac{\omega\left[\mathbf{q}^2+2\mathbf{pq}-2\frac{\varepsilon}{\omega}\mathbf{kq}\right]}{kp-\mathbf{kq}} - (\gamma\mathbf{q})\gamma_0\right](\gamma k)(\gamma\mathbf{D}) + \frac{\omega e\alpha\left(\frac{\mathbf{q}}{kp}\right)(\gamma k)(\gamma\mathbf{A}(\varphi))}{kp-\mathbf{kq}}D_{1,n}(\theta(\mathbf{q})) - \frac{e\omega\left[\mathbf{q}^2+2\mathbf{pq}-2\frac{\varepsilon}{\omega}\mathbf{kq}\right]}{2kp(kp-\mathbf{kq})}(\gamma k)(\gamma\mathbf{A}(\varphi))D_n\Bigg\}u_p^\nu, \quad (13)$$

where $\alpha_1(\mathbf{q})$, $\alpha_2(\mathbf{q})$ are dynamic parameters of the interaction defined by expressions

$$\alpha_1(\mathbf{q}) = \alpha\left(\frac{(\mathbf{kq})\mathbf{p}}{kp}+\mathbf{q}\right), \quad (14)$$

$$\alpha_2(\mathbf{q}) = \frac{\mathbf{kq}}{2(kp-\mathbf{kq})}Z(1-\zeta^2), \quad (15)$$

and $\theta_1(\mathbf{q})$ is the phase angle

$$\theta_1(\mathbf{q}) = \theta\left((\mathbf{kq})\mathbf{p}/kp+\mathbf{q}\right). \quad (16)$$

The functions D_n, $D_{1,n}(\theta(\mathbf{p}))$, $D_{1,n}(\theta(\mathbf{q}))$ and $D_{2,n}$ are defined by relations

$$D_n = J_n(\alpha_1(\mathbf{q}), -\alpha_2(\mathbf{q}), \theta_1(\mathbf{q})), \quad (17)$$

$$D_{1,n}(\theta(\mathbf{p})) = \frac{1}{2}\Big[J_{n-1}(\alpha_1(\mathbf{q}),-\alpha_2(\mathbf{q}),\theta_1(\mathbf{q}))\mathrm{e}^{-\mathrm{i}(\theta_1(\mathbf{q})-\theta(\mathbf{p}))} + J_{n+1}(\alpha_1(\mathbf{q}),-\alpha_2(\mathbf{q}),\theta_1(\mathbf{q}))\mathrm{e}^{\mathrm{i}(\theta_1(\mathbf{q})-\theta(\mathbf{p}))}\Big], \quad (18)$$

$$D_{2,n} = \frac{(1-\zeta^2)}{2}\Big[J_{n-2}(\alpha_1(\mathbf{q}),-\alpha_2(\mathbf{q}),\theta_1(\mathbf{q}))\mathrm{e}^{-\mathrm{i}2\theta_1(\mathbf{q})} + J_{n+2}(\alpha_1(\mathbf{q}),-\alpha_2(\mathbf{q}),\theta_1(\mathbf{q}))\mathrm{e}^{\mathrm{i}2\theta_1(\mathbf{q})}\Big] + (1+\zeta^2)D_n, \quad (19)$$

$$\mathbf{D} \equiv \mathrm{e}\overline{A}_0\left\{\frac{\widehat{\mathbf{e}}_1+\mathrm{i}\zeta\widehat{\mathbf{e}}_2}{2}J_{n-1}(\alpha_1(\mathbf{q}),-\alpha_2(\mathbf{q}),\theta_1(\mathbf{q}))\mathrm{e}^{-\mathrm{i}\theta_1(\mathbf{q})} + \frac{\widehat{\mathbf{e}}_1-\mathrm{i}\zeta\widehat{\mathbf{e}}_2}{2}J_{n+1}(\alpha_1(\mathbf{q}),-\alpha_2(\mathbf{q}),\theta_1(\mathbf{q}))\mathrm{e}^{\mathrm{i}\theta_1(\mathbf{q})}\right\}, \quad (20)$$

where function $J_n(u,v,\Delta)$ is expressed by the series of ordinary Bessel functions $J_k(v)$

$$J_n(u,v,\Delta) = \sum_{k=-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}2k\Delta}J_{n-2k}(u)J_k(v). \quad (21)$$

In the denominator of the integrals in the expressions (9, 10) $-\mathrm{i}0$ is an imaginary infinitesimal, which shows how the path around the pole in the integrand should be chosen to obtain a certain asymptotic behavior of the wave function, *i.e.* the outgoing spherical wave. Note that the wave function is normalized for the one particle in the unit volume.

Let us determine the scattering amplitude by the Green function formalism in GEA (for the elastic scattering see [12]). For the transition amplitude in the EM wave field from the state with the "quasimomentum" Π and the polarization ν to the state with the "quasimomentum" Π' and the polarization μ we have the expression

$$T^{\mu\nu}(\Pi\to\Pi') = \int\overline{\Psi}^V_{\Pi',\mu}(x)\gamma_0\Psi_{\Pi,\nu}(x)U(\mathbf{r})\mathrm{d}^4x, \quad (22)$$

where x is the four-radius vector, $\overline{\Psi} = \Psi^\dagger\gamma_0$ and $\Psi^\dagger$ denotes the transposition and complex conjugation of Ψ. According to (3) and (8) the transition amplitude (22) can be expressed in the following form

$$T^{\mu\nu}(\Pi\to\Pi') = \int\mathrm{e}^{\mathrm{i}(\Pi_0'-\Pi_0)t}B(t,\mathbf{r})\mathrm{d}t, \quad (23)$$

where

$$B(t,\mathbf{r}) = \int\mathrm{e}^{-\mathrm{i}(\Pi_0'-\Pi_0)t}\overline{\Psi}^V_{\Pi',\mu}(x)\gamma_0\Psi_{\Pi,\nu}(x)U(\mathbf{r})\mathrm{d}\mathbf{r} \quad (24)$$

is the periodic function of time. So, making a Fourier transformation of the function $B(t,\mathbf{r})$ over t by the relations

$$B(t,\mathbf{r}) = \sum_{n=-\infty}^{\infty}\widetilde{B}_n\exp(-\mathrm{i}nt), \quad (25)$$

$$\widetilde{B}_n = \frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}B(t,\mathbf{r})\exp(\mathrm{i}nt)\mathrm{d}t, \quad (26)$$

and carrying out the integration over t in the formula (23) we obtain

$$T^{\mu\nu}(\Pi\to\Pi') = 2\pi\widetilde{B}_n\delta\left(\Pi_0'-\Pi_0-n\omega\right). \quad (27)$$

The differential probability of SB process per unit time in the phase space $\mathrm{d}\mathbf{\Pi}'/(2\pi)^3$ (space volume $V=1$ in accordance with normalization of electron wave function) will be

$$\mathrm{d}W_{\Pi\to\Pi'} = \lim_{t\to\infty}\frac{1}{t}\left|T^{\mu\nu}(\Pi\to\Pi')\right|^2|\mathbf{\Pi}'|\,\Pi_0'\mathrm{d}\Pi_0'\frac{\mathrm{d}\Omega}{(2\pi)^3}, \quad (28)$$

where $\mathrm{d}\Omega$ is the differential solid angle.

Dividing the differential probability of SB process (28) by initial flux density $|\mathbf{\Pi}|/\Pi_0$, summing over the particle final states, averaging over initial polarization states and integrating over Π_0' we obtain the differential cross-section of SB process for the non-polarized particles

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_n\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega}, \quad (29)$$

where

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} = \frac{1}{32\pi^2}\frac{|\mathbf{\Pi}'|}{|\mathbf{\Pi}|}\sum_{\mu\nu}\left|\widetilde{B}_n\right|^2\Bigg|_{\Pi_0'=\Pi_0+n\omega} \quad (30)$$

is the partial differential cross-section which describes n-photon SB process. Because of very complicated analytical expressions for multiphoton cross-sections of SB in GEA being considered, the ultimate results require numerical investigations that will be presented elsewhere.

Now let us proceed to the asymptotic approach to construct the multiphoton cross-sections of SB. Note that at asymptotic large distances $r \to +\infty$ the GEA wave function coincides with the Born approximation one when $|S_1(\mathbf{r},\, t)| \ll 1$ [13]. As far as we consider an inelastic scattering, the wave function of the particle at large distances $r \to +\infty$ will be the sum of spherical convergent waves with the superposition of a plane wave

$$\lim_{r\to+\infty} \Psi(\mathbf{r},\, t) = u_p^{\nu} \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{r}-\mathrm{i}\varepsilon_0 t} + \sum_{n=-\infty}^{\infty} G_n(\widehat{\mathbf{r}}) \frac{\mathrm{e}^{\mathrm{i}|\boldsymbol{\Pi}_n|r-\mathrm{i}\Pi_n t}}{r}, \tag{31}$$

where $G_n(\widehat{\mathbf{r}})$ is a bispinor depending on $\widehat{\mathbf{r}} = \mathbf{r}/r$. Each term of sum describes n photon SB process and the partial inelastic scattering amplitude will be defined as

$$f_n^{\mu\nu}(\boldsymbol{\Pi} \to \boldsymbol{\Pi}') = \frac{1}{2m} \overline{u}_{p'}^{\mu} G_n(\widehat{\mathbf{r}}), \tag{32}$$

and for the partial differential cross-section of SB for the non-polarized particles we have

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} = \frac{1}{2} \frac{|\boldsymbol{\Pi}'|}{|\boldsymbol{\Pi}|} \sum_{\mu\nu} \left| f_n^{\mu\nu}(\boldsymbol{\Pi} \to \boldsymbol{\Pi}') \right|^2. \tag{33}$$

The wave function of the particle for SB process in GEA (8) in asymptotic limit of large r has the form

$$\lim_{r\to+\infty} \Psi(\mathbf{r}, t) = \lim_{r\to+\infty} \frac{1}{\sqrt{2\Pi_0}} \exp\left(\mathrm{i}S_{\mathrm{V}}(x)\right) \times \left\{ f_{\mathrm{V}}(\varphi) + \frac{\mathrm{e}^{-\mathrm{i}\boldsymbol{\Pi}\cdot\mathbf{r}}}{4\pi r} \sum_{n\,=-\,n_m}^{\infty} \mathrm{e}^{\mathrm{i}(\Pi_n \widehat{\mathbf{r}} - n\mathbf{k})\cdot\mathbf{r}} F_n\left(\varphi, \mathbf{q}_n\right) \right\}, \tag{34}$$

where

$$\mathbf{q}_n = \Pi_n \widehat{\mathbf{r}} - \boldsymbol{\Pi} - n\mathbf{k}, \tag{35}$$

is the momentum that ion compensates and

$$\Pi_n = \sqrt{\boldsymbol{\Pi}^2 + n\omega\left(2\Pi_0 + n\omega\right)}, \tag{36}$$

is the final quasimomentum of the particle corresponding to n-photon absorption ($n > 0$) and emission ($n < 0$) processes. In (34) n_m is the maximum number of emitted photons:

$$n_m = \frac{\Pi_0 - m_*}{\omega}, \tag{37}$$

where

$$m_* = m\sqrt{1 + K_0^2 \frac{(1+\zeta^2)}{2}} \tag{38}$$

is the "effective mass" of the electron in the EM wave field and

$$K_0 = \frac{e\overline{A}_0}{m} \tag{39}$$

is the relativistic invariant parameter of intensity. The bispinor function $F_n\left(\varphi, \mathbf{q}_n\right)$ is defined by relation (13) at the $r \to +\infty$. Then from the equations (32, 34) follows that the bispinor $G_n(\widehat{\mathbf{r}})$ coincides with the function $F_n(r \to +\infty, \mathbf{q})$ (the unessential phase corrections are neglected):

$$\begin{aligned} G_n(\widehat{\mathbf{r}}) &= F_n(r \to +\infty, \mathbf{q}_n) = \frac{1}{4\pi} \widetilde{U}(\mathbf{q}_n) \Big\{ D_n(\gamma\mathbf{q}_n)\gamma_0 \\ &+ Z D_{2,n} \frac{(\mathbf{k}\mathbf{q}_n)(\gamma k)\gamma_0 - \omega(\gamma k)(\gamma\mathbf{q}_n)}{kp - \mathbf{k}\cdot\mathbf{q}_n} + \frac{(\gamma k)(\gamma\mathbf{D})(\gamma\mathbf{q}_n)\gamma_0}{2(kp - \mathbf{k}\mathbf{q}_n)} \\ &+ \frac{1}{2kp} \left[\frac{\omega\left[\mathbf{q}_n^2 + 2\mathbf{p}\mathbf{q}_n - 2\frac{\varepsilon}{\omega}\mathbf{k}\mathbf{q}_n\right]}{kp - \mathbf{k}\mathbf{q}_n} - (\gamma\mathbf{q}_n)\gamma_0 \right] (\gamma k)(\gamma\mathbf{D}) \\ &- 2\left[\varepsilon D_n - \omega\alpha\left(\frac{\mathbf{p}}{kp}\right) D_{1,n}(\theta(\mathbf{p})) + \omega Z D_{2,n} \right] \Big\} u_p^{\nu}. \end{aligned} \tag{40}$$

Here the functions $D_n, D_{1,n}, D_{2,n}$ and $\mathbf{D}$ are defined by the expressions (17–20), and

$$\alpha_1(\mathbf{q}_n) = \alpha_1\left(\frac{\mathbf{p}'}{kp'} - \frac{\mathbf{p}}{kp}\right) = \alpha_1\left(\frac{\boldsymbol{\Pi}'}{k\Pi'} - \frac{\boldsymbol{\Pi}}{k\Pi}\right), \tag{41}$$

$$\alpha_2(\mathbf{q}_n) = \frac{Z' - Z}{2}(1 - \zeta^2), \tag{42}$$

$$\theta_1(\mathbf{q}_n) = \theta\left(\frac{\mathbf{p}'}{kp'} - \frac{\mathbf{p}}{kp}\right) = \theta\left(\frac{\boldsymbol{\Pi}'}{k\Pi'} - \frac{\boldsymbol{\Pi}}{k\Pi}\right)\cdot \tag{43}$$

In addition, taking into account that

$$\overline{u}_{p'}(p_0'\gamma_0 - \gamma\mathbf{p}' - m) = 0,$$
$$(p_0\gamma_0 - \gamma\mathbf{p} - m)u_p = 0,$$

and known relations between the γ-matrix elements

$$\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu} = 2\delta^{\mu\nu},$$

we reduce the transition amplitude to the following form

$$f_n^{\mu\nu}(\boldsymbol{\Pi} \to \boldsymbol{\Pi}') = -\frac{1}{4\pi} \overline{u}_{p'}^{\mu} Q u_p^{\nu} \widetilde{U}(\mathbf{q}_n), \tag{44}$$

where

$$Q = \gamma_0 D_n + \frac{\omega Z}{kp'}(\gamma k) D_{2,n} - (\gamma k')(\gamma\mathbf{D}), \tag{45}$$

$$k' = \left(\frac{k}{2kp} + \frac{\widetilde{k}}{2kp'}\right), \quad \widetilde{k} = \left(k_0, -\mathbf{k}\right). \tag{46}$$

Then, introducing (44) into the (33), we will write the scattering cross-section in the form

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} = \frac{|\boldsymbol{\Pi}'| \left|\widetilde{U}(\mathbf{q}_n)\right|^2}{(4\pi)^2 |\boldsymbol{\Pi}|} 2Sp\left\{\rho' Q \rho \overline{Q}\right\}, \tag{47}$$

where

$$\rho = \frac{\gamma p + m}{2},\ \rho' = \frac{\gamma p' + m}{2}$$

are the initial and final density matrices.

Taking into account that $kp = k\Pi$, $kp' = k\Pi'$ and using the properties of defined functions $D_n, D_{1,n}, D_{2,n}$ and $\mathbf{D}$ which follow directly from equations (17–20) and recurrent relation for functions $J_n(u,v,\Delta)$

$$2nJ_n(u,v,\Delta) = u\left[J_{n-1}(u,v,\Delta) + J_{n+1}(u,v,\Delta)\right] + 2v\left[\mathrm{e}^{-\mathrm{i}2\Delta}J_{n-2}(u,v,\Delta) + \mathrm{e}^{\mathrm{i}2\Delta}J_{n+2}(u,v,\Delta)\right], \quad (48)$$

we obtain the following expression for the partial differential cross-sections of SB process

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} = \frac{|\mathbf{\Pi}'|}{(4\pi)^2\,|\mathbf{\Pi}|}\left|\widetilde{U}(\mathbf{q}_n)\right|^2\Bigg\{\left[4\varepsilon D_n + \omega Z D_{2,n} - \omega\alpha\left(\frac{\mathbf{\Pi}}{k\Pi}\right)D_{1,n}(\theta(\mathbf{\Pi}))\right]^2 - \mathbf{q}_n^2\,|D_n|^2 + \frac{1}{(k\Pi)(k\Pi')}\left[\omega^2\mathbf{q}_n^2 - (\mathbf{k}\mathbf{q}_n)^2\right] \times \left(|\mathbf{D}|^2 - \frac{e^2\overline{A}_0^2}{2}\mathrm{Re}\left(D_n D_{2,n}^{\dagger}\right)\right)\Bigg\}, \quad (49)$$

where

$$|\mathbf{D}|^2 = \frac{e^2\overline{A}_0^2}{4}\Big\{\left(1+\zeta^2\right)\left(|J_{n-1}|^2 + |J_{n+1}|^2\right) + 2\left(1-\zeta^2\right)\Big[\cos 2\theta_1(\mathbf{q}_n)\mathrm{Re}\left(J_{n-1}J_{n+1}^{\dagger}\right) + 2\mathrm{i}\sin 2\left(\theta_1(\mathbf{q}_n) - \theta(\mathbf{p})\right)\mathrm{Im}\left(J_{n-1}J_{n+1}^{\dagger}\right)\Big]\Big\}\cdot \quad (50)$$

Comparing the cross-sections of SB process for spinor and scalar particles we conclude that the spin interaction is described by the terms which are in order of square of the quantum recoil $\sim \mathbf{q}_n^2$ (last two terms in figurate brackets). Hence, the spin interaction gives a considerable contribution in the SB differential cross-section only for the large-angle scattering (which is known also for the elastic scattering from the formula of Mott) and for the relativistic intensities of EM wave at $K_0 \gg 1$.

3 Differential cross-sections of SB on the screening Coulomb potential for the circular and linear polarizations of EM wave

In particular case we utilize the equation (49) in order to obtain the differential cross-section of SB on a screening Coulomb potential for which the Fourier transform is

$$\widetilde{U}(\mathbf{q}_n) = \frac{4\pi Z_a e^2}{\mathbf{q}_n^2 + \chi^2}, \quad (51)$$

where $1/\chi$ is the radius of screening, Z_a is the charge number of the nucleus.

For circular polarized EM wave the quantities in (49) are given by relations (41–43, 17–20) at $\zeta = 1$. Then, taking into account that at $\alpha_2(\mathbf{q}) = 0$ the functions D_n, $D_{1,n}(\theta(\mathbf{\Pi}))$ and $D_{2,n}$ are expressed by ordinary Bessel functions, for the partial differential cross-section of SB we have

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} = \frac{(Z_a e^2)^2\,|\mathbf{\Pi}'|}{|\mathbf{\Pi}|\,(\mathbf{q}_n^2 + \chi^2)^2}\Bigg\{J_n^2(\alpha_1(\mathbf{q}_n)) \times \left[4\left(\Pi_0 - \frac{n\omega\alpha\left(\frac{\mathbf{\Pi}}{k\Pi}\right)}{\alpha_1(\mathbf{q}_n)}\cos\left[\theta_1(\mathbf{q}_n) - \theta(\mathbf{p})\right]\right)^2 - \mathbf{q}_n^2 + \beta^2\left(\frac{n^2}{\alpha_1^2(\mathbf{q}_n)} - 1\right)\right] + J_n'^2(\alpha_1(\mathbf{q}_n)) \times \left(4\omega^2\alpha^2\left(\frac{\mathbf{p}}{kp}\right)\sin^2\left[\theta_1(\mathbf{q}_n) - \theta(\mathbf{p})\right] + \beta^2\right)\Bigg\}, \quad (52)$$

where

$$\beta^2 = \frac{e^2\overline{A}_0^2}{(kp)(kp')}\left[\omega^2\mathbf{q}_n^2 - (\mathbf{k}\cdot\mathbf{q}_n)^2\right]. \quad (53)$$

and $J_n'(x)$ denotes the first derivative of ordinary Bessel function with respect to x.

In the case of linearly polarized EM wave all quantities are defined for $\zeta = 0$. As far as $\theta_1(\mathbf{q}_n) = \theta(\mathbf{p}) = 0$, the functions D_n, $D_{1,n}(\theta(\mathbf{\Pi}))$ and $D_{2,n}$ are defined by the real function $J_n(u,v)$ [15], further called the generalized Bessel function [16]. Making the simple transformations and using the expression (48) we obtain the partial differential cross-section of SB process $\mathrm{d}\sigma^{(n)}/\mathrm{d}\Omega$ for the linear polarization of the wave

$$\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} = \frac{(Z_a e^2 m)^2\,|\mathbf{\Pi}'|}{|\mathbf{\Pi}|\,|\mathbf{q}_n^2 + \chi^2|^2} \times \Bigg\{J_n^2(u,v)\left[\epsilon^2 - \mathbf{q}_n^2 - \beta^2\left(\frac{1}{2} + \frac{n}{4v}\right)\right] + I_n^2(u,v)\left[4\omega^2\alpha'^2 + \beta^2\right] + J_n(u,v)I_n(u,v)\left[\frac{u\beta^2}{4v} - 4\omega\epsilon\alpha'\right]\Bigg\}, \quad (54)$$

where

$$I_n(u,v) = \frac{1}{2}\left(J_{n-1}(u,v) + J_{n+1}(u,v)\right),$$

and

$$\epsilon = 2\Pi_0 + \frac{n\omega Z}{v}, \quad (55)$$

$$\alpha' = \alpha\left(\frac{\mathbf{\Pi}}{k\Pi}\right) + \frac{uZ}{2v}\cdot \quad (56)$$

The arguments u, v are determined by the relations

$$u = e\overline{\mathbf{A}}_0 \cdot \left(\frac{\mathbf{\Pi}'}{k\Pi'} - \frac{\mathbf{\Pi}}{k\Pi} \right), \quad (57)$$

$$v = -\alpha_2(\mathbf{q}_n) = \frac{Z - Z'}{2} \cdot \quad (58)$$

Comparing the non relativistic cross-section [5] with the relativistic one, it is easy to see that, besides the additional terms, which result from spin-orbital and spin-laser interaction ($\sim \mathbf{q}_n^2$) as well as from the intensity ($\sim K_0^2$), the relativistic contribution is conditioned by arguments of the Bessel functions. Because of sensitivity of the Bessel function to relationship of it's argument and index the most probable number of emitted or absorbed photons will be defined by the condition $|n| \sim |\alpha_1(\mathbf{q}_n)|$. By this reason the contribution of relativistic effects to the scattering process, as shown in reference [8], becomes essential already for $K_0 \sim 0.1$. Consequently, the dipole approximation is violated for nonrelativistic parameters of interaction. Besides, the state of an electron in the field of a strong EM wave and consequently the cross-section of SB essentially depends on the polarization of the wave. Thus, for the circular polarization the relativistic parameter of the wave intensity $K^2 = const = K_0^2$ and the longitudinal velocity of the electron in the wave $v_{II} = const$, meanwhile for the linear one $K^2 = K_0^2 \cos^2 \varphi$ and v_{II} oscillates with the frequencies of all wave harmonics $n\omega$ corresponding to strongly unharmonic oscillatory motion of an electron. The later leads to different behavior of SB process particularly cross-section for linear polarization of the wave is described by the generalized Bessel function. The cross-sections in both cases are complicated and for the analysis we have performed numerical investigation.

For all numerical calculations we have chosen the initial electron momentum $\mathbf{p}$ to be colinear with laser propagation direction. In this case for circular polarization of wave there is an azimuthal symmetry with respect to propagation direction which simplifies the calculation of integral quantities. For the numerical simulations we have taken moderate initial electron kinetic energy $\varepsilon_k = 2.7$ keV (100 a.u.), Neodymium laser ($\hbar\omega \simeq 1.17$ eV), radius of screening $\chi^{-1} = 4$ a.u. and $Z_a = 1$.

In Figure 1 the numerical calculations are presented for the same parameters as they were presented in reference [8] for the circular polarization of EM wave. In Figure 1a, the envelopes of partial differential cross-sections are shown for circular polarization of EM wave as a function of number of emitted or absorbed photons for the deflection angle $\vartheta \equiv \angle \mathbf{\Pi\Pi}' = 0.6$ mrad. The laser intensity is taken to be 3.5×10^{16} W/cm^2, which corresponds to relativistic parameter of intensity $K \simeq 0.17$, or electric field strength $E = 1$ a.u. The doted and dashed lines correspond to initial electron momentum parallel and antiparallel to the laser propagation direction $\mathbf{k}$ respectively, and the solid line gives the nonrelativistic result. In Figure 1b the envelopes of partial differential cross-sections for linear polarization of EM wave are shown. To emphasize the differences, the relativistic parameter of intensity is taken the same. For linear polarization of the wave there is no azimuthal symmetry and we have taken the final electron momentum in the same plane with $\mathbf{A}$ and $\mathbf{k}$. The deflection angle is again 0.6 mrad.

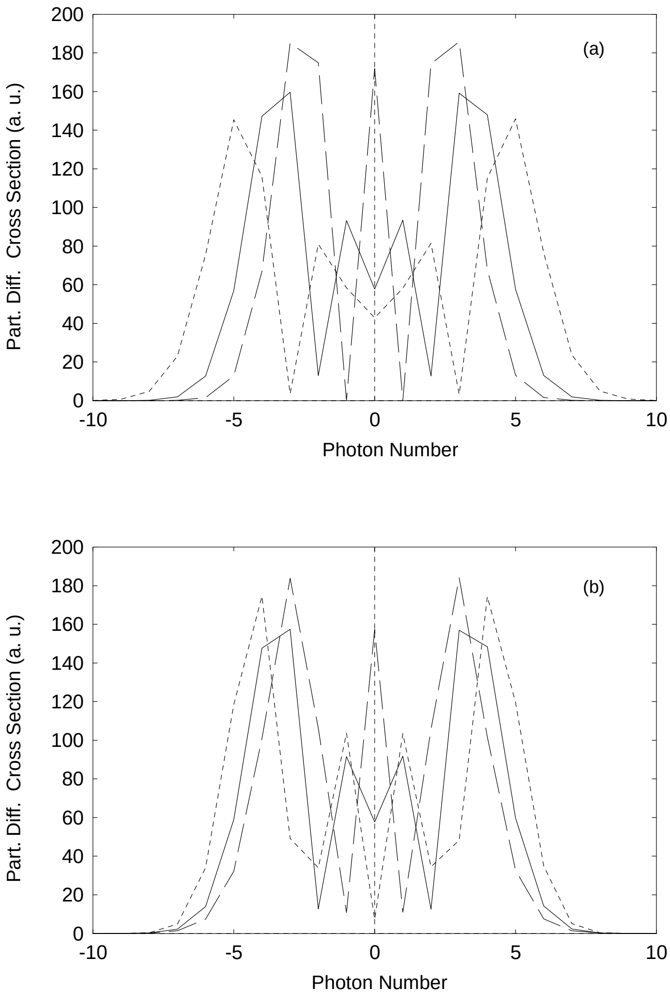

Fig. 1. The envelopes of partial differential cross-sections $\mathrm{d}\sigma^{(n)}/\mathrm{d}\Omega$ in atomic units as a function of the number of emitted or absorbed photons, for an intensity of neodymium laser 3.5×10^{16} W/cm^2 ($K \simeq 0.17$): (a) and (b) correspond to circular and linear polarization of EM wave respectively. The deflection angle equals $\vartheta = 0.6$ mrad. The doted and dashed lines correspond to initial electron momentum parallel and antiparallel to the laser propagation direction $\mathbf{k}$ respectively, and the solid line gives the nonrelativistic result.

The energy exchange increases for large deflection angles. In Figure 2a the envelopes of partial differential cross-sections as a function of the number of emitted or absorbed photons for circular polarization of EM wave are shown for the large deflection angle $\vartheta = 10$ mrad. The relativistic parameter of intensity is taken to be $K_0 \simeq 0.1$. The energy change of a particle is characterized by the absorption/emission (AE) cross-section. Partial AE differential cross-section will be

$$\frac{\mathrm{d}\sigma_{\mathrm{ae}}^{(n)}}{\mathrm{d}\Omega} = n \left(\frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} - \frac{\mathrm{d}\sigma^{(-n)}}{\mathrm{d}\Omega} \right) \cdot \quad (59)$$

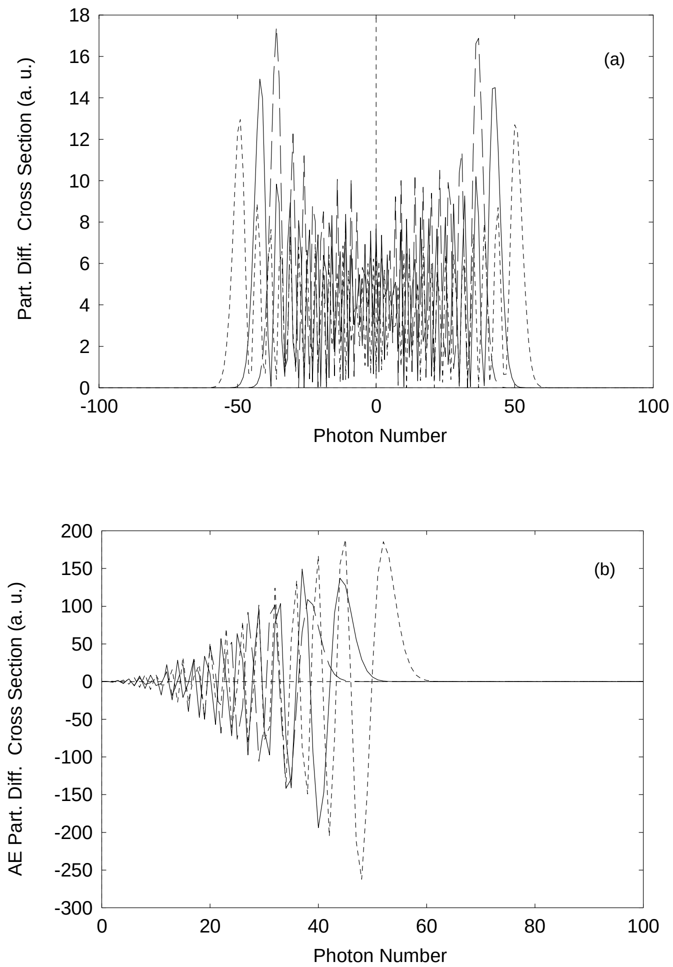

Fig. 2. Part (a) displays envelopes of partial differential cross-sections as a function of the number of emitted or absorbed photons for circular polarization of EM wave for the large deflection angle $\vartheta = 10$ mrad. The relativistic parameter of intensity is $K_0 = 0.1$. In (b) the envelopes of partial absorption/emission differential cross-sections are shown for the same parameters.

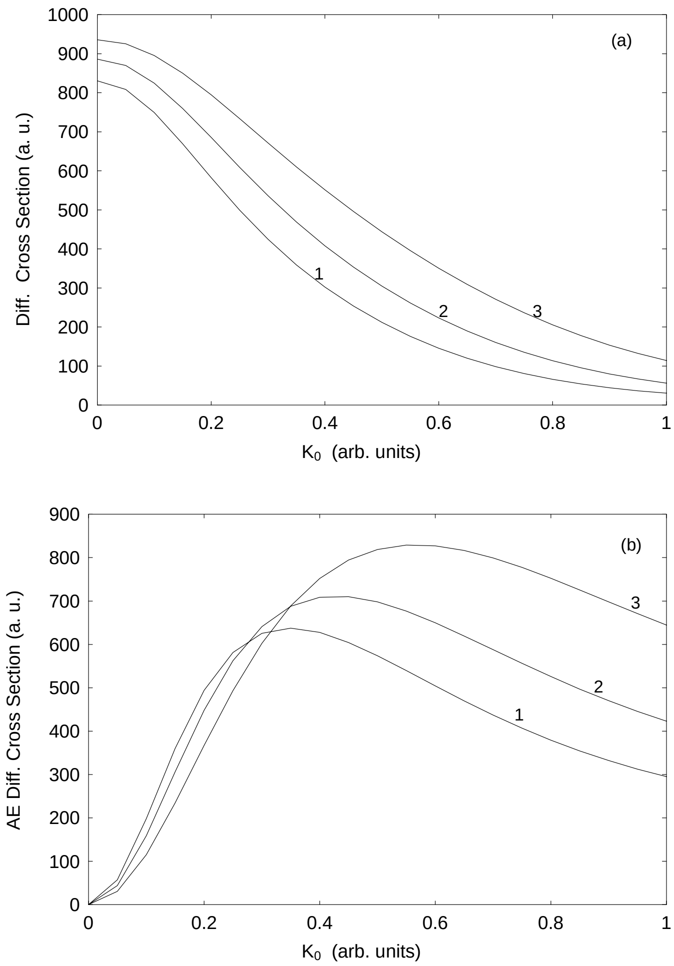

Fig. 3. The summed differential cross-sections for circular polarization of EM wave are plotted as a function of relativistic parameter of intensity K_0 in the range $0 \le K_0 \le 1$. The initial electron momentum is parallel to the laser propagation direction $\mathbf{k}$. Part (a) displays SB differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$, (b) shows absorption/emission differential cross-section $\mathrm{d}\sigma_{\mathrm{ae}}/\mathrm{d}\Omega$. Numbers denote different values of deflection angle: 1, $\vartheta = 6$ mrad; 2, $\vartheta = 5$ mrad; 3, $\vartheta = 4$ mrad.

In Figure 2b the envelopes of partial AE differential cross-sections for circular polarization of EM wave are shown for the same parameters. As it is seen from Figures 1 and 2, the differences between the cases of initial electron momentum parallel or antiparallel to the laser propagation direction $\mathbf{k}$ on the one hand and between nonrelativistic result on the other hand are notable already for $K_0 \simeq 0.1$. Particularly the absorption and emission edges and the magnitudes of the peaks are different.

To show the dependence of SB process upon laser intensity in the Figure 3a the summed differential cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sum_{n=-n_m}^{\infty} \frac{\mathrm{d}\sigma^{(n)}}{\mathrm{d}\Omega} \tag{60}$$

is plotted for various deflection angles as a function of relativistic parameter of intensity K_0. The initial electron momentum is parallel to the laser propagation direction $\mathbf{k}$. In Figure 3b summed AE differential cross-section is shown. We see from Figure 3 that as SB as well as AE cross-sections decrease with the increasing of the wave intensity. This is a consequence of the SB process being essentially nonlinear in contrast with perturbation theory where n-photon SB cross-section $\sim K_0^{2n}$. To show the angular distribution of scattered electrons in the Figure 4 we display the angular dependence of the summed differential cross-sections for various K_0. The initial electron momentum is parallel to the laser propagation direction $\mathbf{k}$. For comparison we have also plotted the angular distribution of Mott scattering ($K_0 = 0$). As we see the angular distribution becomes narrower with the increasing of the wave intensity. For the integral quantities such as the total scattering cross-section σ and total emission/absorption cross-section (σ_{T}) which characterizes net energy change one should integrate partial differential cross-section of SB process $\mathrm{d}\sigma^{(n)}/\mathrm{d}\Omega$ over solid angle and make summation

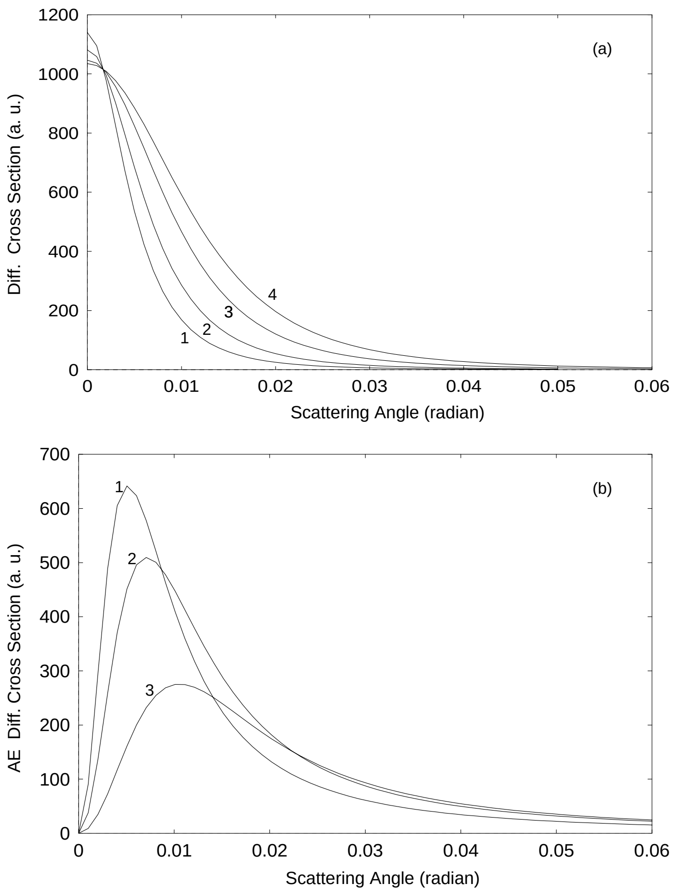

Fig. 4. The angular dependence of SB process for various laser intensities at circular polarization of EM wave. The initial electron momentum is parallel to the laser propagation direction $\mathbf{k}$. Part (a) displays summed differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$ as a function of deflection angle ϑ. Numbers denote different values of relativistic parameter of intensity: 1, $K_0 = 0.3$; 2, $K_0 = 0.2$; 3, $K_0 = 0.1$; 4 $K_0 = 0$ (Mott scattering). In (b) with the same labeling the absorption/emission differential cross-section $\mathrm{d}\sigma_{\mathrm{ae}}/\mathrm{d}\Omega$ is shown.

over photon numbers:

$$\sigma = \sum_{n=-n_m}^{\infty} \sigma^{(n)}, \tag{61}$$

and total AE cross-section (σ_{T}) will be

$$\sigma_{\mathrm{T}} = \sum_{n=0}^{\infty} \sigma_{\mathrm{ae}}^{(n)}. \tag{62}$$

Note that for these quantities in the optical range of frequencies one can neglect the contribution which comes from the spin interaction. The latter is essential for large angle scatterings which give minor contribution to the total cross-sections (for optical frequencies the quantum recoil is negligibly small). For the strong laser fields one should take into account a large number of terms in (61, 62) since multiphoton absorption/emission processes play significant role already for moderate laser intensities ($K_0 \ll 1$) in contrast for example to nonlinear

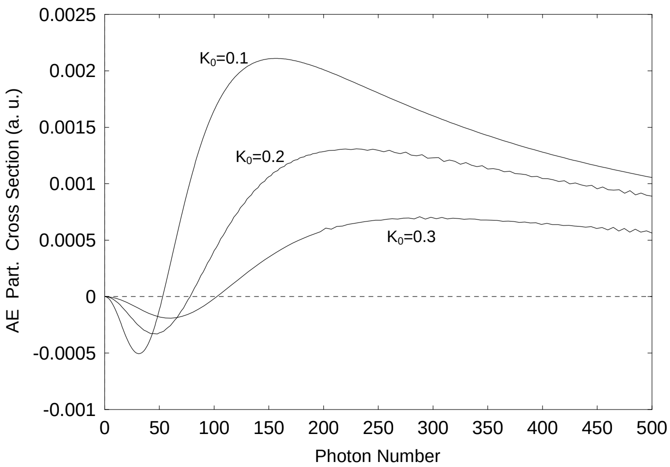

Fig. 5. The envelopes of integrated absorption/emission partial cross-sections $\sigma_{\mathrm{ae}}^{(n)}$ for circular polarization of EM wave as a function of photon number in the range $0 \leq n \leq 500$ for various laser intensities. The initial electron momentum parallel to the laser propagation direction $\mathbf{k}$. Negative values correspond to net emission, while positive values correspond to net absorption.

Compton scattering [15] were multiphoton processes become essential for $K_0 \sim 1$ and the cutoff number of absorbed photons $\sim K_0^3$. This essentially complicates the analysis of total cross-sections (61, 62). As a first step to exhibit the dependence of SB process upon laser intensity in Figure 5 the envelopes of integrated AE partial cross-sections $\sigma_{\mathrm{ae}}^{(n)}$ for various laser intensities as a function of the photon number in the range $0 \leq n \leq 500$ are plotted. The initial electron momentum is parallel to the laser propagation direction $\mathbf{k}$. Negative values correspond to net emission, while positive values correspond to net absorption. From the Figure 5 we see that for this initial geometry the absorption process is dominant but with the increasing of the wave intensity the AE cross-section reduces.

4 Conclusion

We have presented the theoretical treatment of the multiphoton stimulated bremsstrahlung process. On the base of time dependent relativistic generalized eikonal approximation wave function the multiphoton cross-sections of a Dirac particle bremsstrahlung on an arbitrary electrostatic potential and strong laser radiation field have been investigated. In the limit of the Born approximation relatively simple analytical formulas for SB at arbitrary polarization of EM wave have been presented. The numerical analyses show that SB in strong laser fields is essentially nonlinear, the multiphoton absorption/emission processes and relativistic effects play significant role already for moderate laser intensities. Although our analyses are limited by a given initial geometry we can conclude that as SB as well as AE cross-sections have a tendency to fall with the increasing of wave intensity.

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